Bode Plot Problem

Problem

Given the following loop gain:

$$T(s) = \frac{A(1+\frac{s}{\omega_1})}{(1+\frac{s}{Q\omega_2}+\frac{s^2}{\omega_2^2})}$$

where

$$A = 50$$
$$\omega_1 = 150 rds/s$$
$$\omega_2 = 8 rds/s$$
$$Q = 3$$

Using asymptotic approximations only,

a) Sketch the Bode magnitude and phase plots (~~on the following blank page~~). Be sure to label all break frequencies, slopes of sloping line, gains of sloping lines and gain and phase levels on zero slope lines.

b) Find the maximum gain (as an absolute value) and the frequency or range of frequencies at which it occurs.

c) Using your plots determine the phase margin and associated crossover frequency.

d) Using your plots determine the gain margin and associated crossover frequency.

e) Determine whether the closed loop system is stable.

f) With the assumption that ω_1 may be moved, what value should it take to achieve a 45° phase margin.

SOLUTION

a)

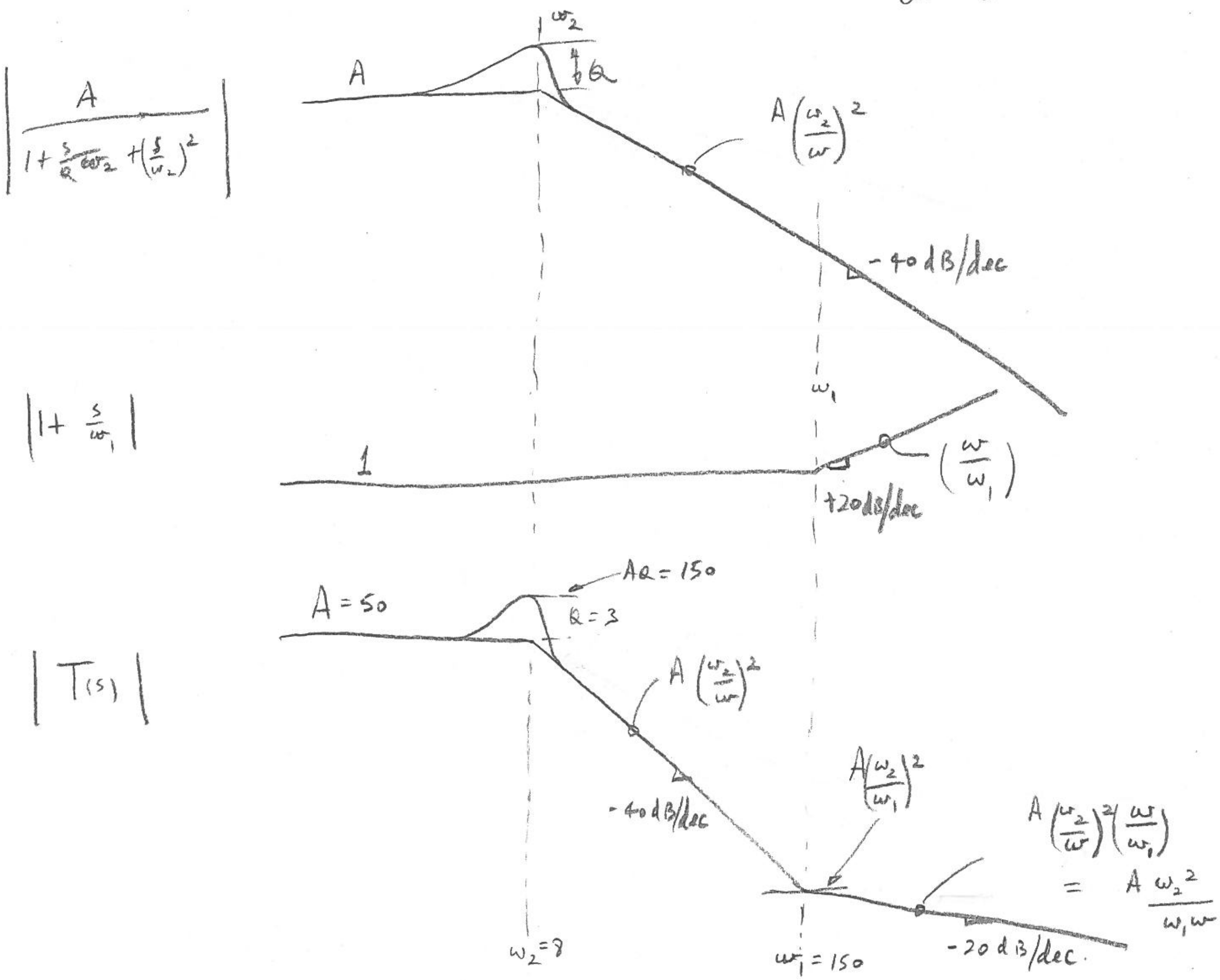
$$T(s) = \frac{A\left(1+\frac{s}{\omega_1}\right)}{1+\frac{s}{Q\omega_2}+\left(\frac{s}{\omega_2}\right)^2}$$

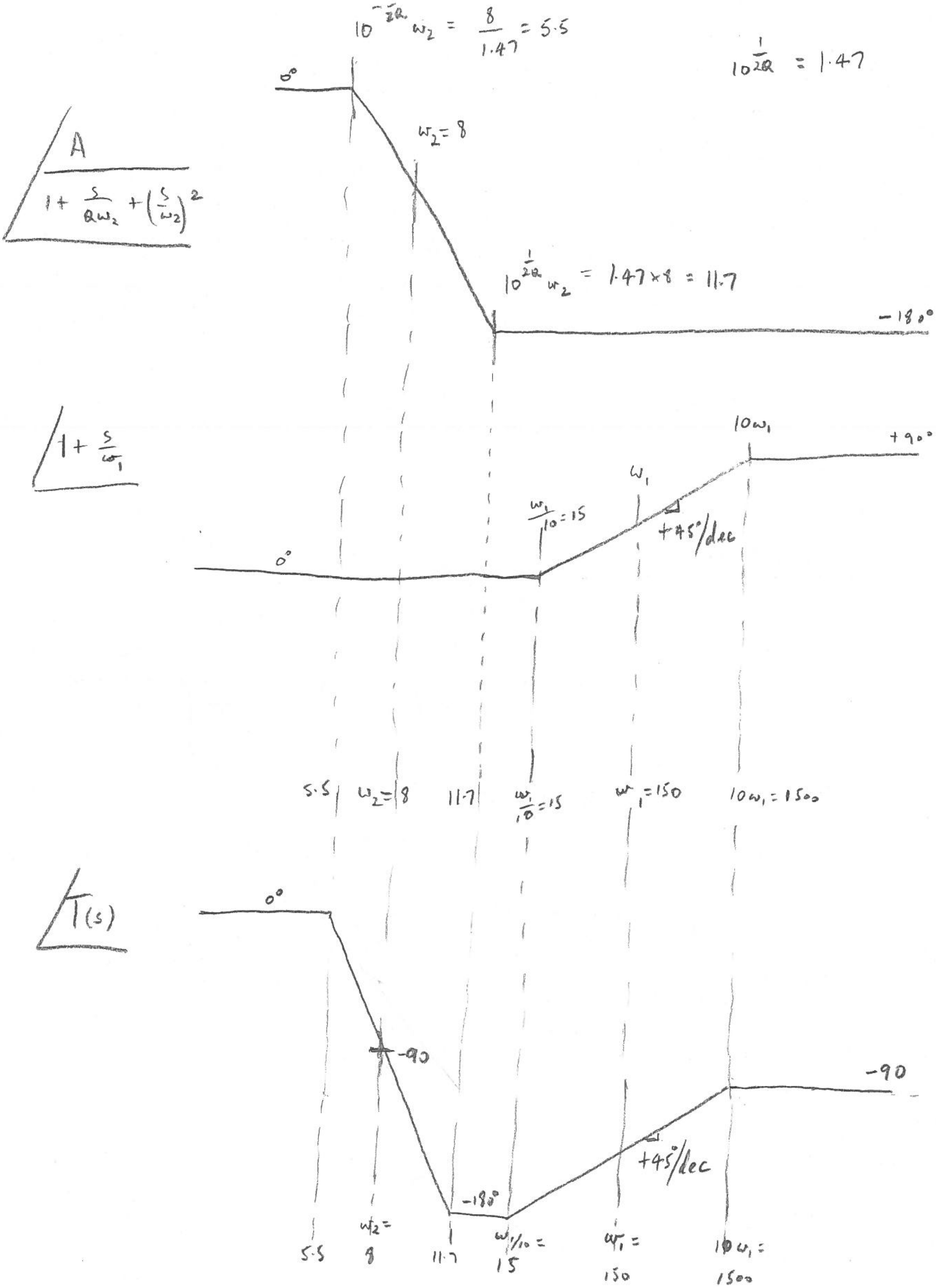
$A = 50$

$\omega_1 = 150$

$\omega_2 = 8$

$Q = 3$





$10^{-\frac{1}{2Q}}\omega_2 = \frac{8}{1.47} = 5.5$
$10^{\frac{1}{2Q}} = 1.47$
$\angle \frac{A}{1+\frac{s}{Q\omega_2}+\left(\frac{s}{\omega_2}\right)^2}$
0°
$\omega_2 = 8$
$10^{\frac{1}{2Q}}\omega_2 = 1.47 \times 8 = 11.7$
−180°
$\angle 1+\frac{s}{\omega_1}$
$10\omega_1$
+90°
ω_1
$\frac{\omega_1}{10} = 15$
+45°/dec
0°
5.5
$\omega_2 = 8$
11.7
$\frac{\omega_1}{10} = 15$
$\omega_1 = 150$
$10\omega_1 = 1500$
$\angle T(s)$
0°
−90
−90
+45°/dec
−180°
5.5
$\omega_2 = 8$
11.7
$\omega_1/10 = 15$
$\omega_1 = 150$
$10\omega_1 = 1500$

$$T(s) = \frac{A\left(1+\frac{s}{\omega_1}\right)}{1+\frac{s}{Q\omega_2}+\left(\frac{s}{\omega_2}\right)^2}$$

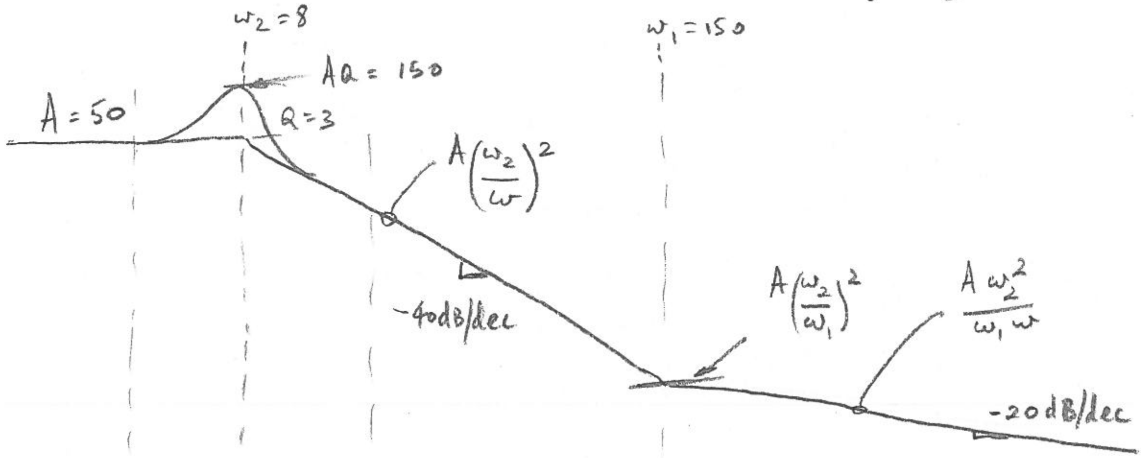
$A = 50$

$\omega_1 = 150$

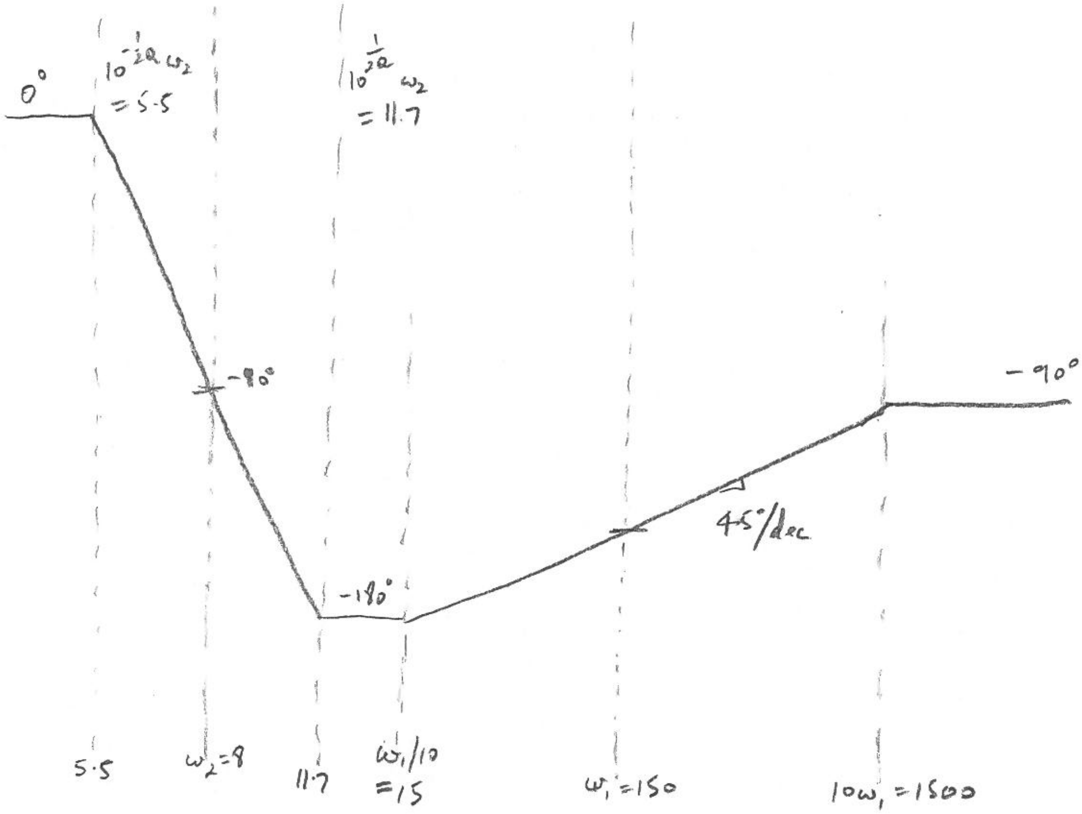
$\omega_2 = 8$

$Q = 3$

$|T(s)|$



$\angle T(s)$



b) MAX. GAIN OCCURS AT $\omega_2 = 8$ AND IS GIVEN BY $QA = 3 \times 50 = 150$

c) AT $\omega_1 = 150$ GAIN IS $A\left(\frac{\omega_2}{\omega_1}\right)^2 = 50\left(\frac{8}{150}\right) = 0.14 < 1$

$\Rightarrow$ UNITY GAIN OCCURS AT $\omega_C < \omega_1$

$$\Rightarrow \quad A\left(\frac{\omega_2}{\omega_C}\right)^2 = 1 \quad \Rightarrow \quad \omega_C = \omega_2\sqrt{A} = 8\sqrt{10} = 56.6 \text{ rds/s}$$

EXACT PHASE ϕ IS GIVEN BY

$$\phi = \underbrace{-\tan^{-1}\left[\frac{\frac{1}{Q}\frac{\omega}{\omega_2}}{1-\left(\frac{\omega}{\omega_2}\right)^2}\right]}_{\approx -180^\circ \text{ for } \omega > 11.7 \text{ rds/s (see phase plot)}} + \tan^{-1}\left(\frac{\omega}{\omega_1}\right), \quad \forall \omega$$

$$\Rightarrow \quad \phi_C \approx -180^\circ + \tan^{-1}\left(\frac{\omega_C}{\omega_1}\right)$$

$$\approx -180^\circ + \tan^{-1}\left(\frac{56.6}{150}\right)$$

$$\approx -160^\circ$$

$\Rightarrow \phi_m =$ PHASE MARGIN $= 180^\circ - 160^\circ$

$= 20^\circ$

d) ∞ GAIN MARGIN, NO ASSOCIATED FREQUENCY

e) $\phi_m > 0 \Rightarrow$ STABLE

f) FROM ABOVE WE SAW

$$\phi_C \approx -180^\circ + \tan^{-1}\left(\frac{\omega_C}{\omega_1}\right)$$

$$\approx -180 + \tan^{-1}\left(\frac{\omega_2\sqrt{A}}{\omega_1}\right)$$

PHASE MARGIN

$$\phi_m = 45^\circ \Rightarrow \phi = -135^\circ$$

$$\Rightarrow \quad -135^\circ = -180 + \tan^{-1}\left(\frac{\omega_2\sqrt{A}}{\omega_1}\right)$$

$$\Rightarrow \quad \omega_1 = \frac{\omega_2\sqrt{A}}{\tan 45}$$

$$= \omega_2\sqrt{A}$$

$$= 56.6 \text{ rds/s.}$$